

ENGINEERING OPTIMIZATION

**Theory and Practice
Third Edition**

SINGIRESU S. RAO

School of Mechanical Engineering
Purdue University
West Lafayette, Indiana



A Wiley-Interscience Publication

John Wiley & Sons, Inc.

New York • Chichester • Brisbane • Toronto • Singapore

This text is printed on acid-free paper.

Copyright © 1996 by John Wiley & Sons, Inc., Wiley Eastern Limited, Publishers, and New Age International Publishers, Ltd.

All rights reserved. Published simultaneously in Canada.

Reproduction or translation of any part of this work beyond that permitted by Section 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful. Requests for permission or further information should be addressed to the Permissions Department, John Wiley & Sons, Inc.

This publication is designed to provide accurate and authoritative information in regard to the subject matter covered. It is sold with the understanding that the publisher is not engaged in rendering professional services. If legal, accounting, medical, psychological, or any other expert assistance is required, the services of a competent professional person should be sought.

Library of Congress Cataloging in Publication Data:

ISBN 0-471-55034-5

Printed in the United States of America

10 9 8 7 6 5 4

PREFACE

The ever-increasing demand on engineers to lower production costs to withstand competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products both economically and efficiently. Optimization techniques, having reached a degree of maturity over the past several years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems being solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a simple manner. Essential proofs and explanations of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using nonlinear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. This book presents the theory and applications of all optimization techniques in a comprehensive manner. Some of the recently developed methods of optimization, such as genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy optimization, are also discussed in the book.

A large number of solved examples, review questions, problems, figures, and references are included to enhance the presentation of the material. Al-

though emphasis is placed on engineering design problems, examples and problems are taken from several fields of engineering to make the subject appealing to all branches of engineering.

This book can be used either at the junior/senior or first-year-graduate-level optimum design or engineering optimization courses. At Purdue University, I cover Chapters 1, 2, 3, 5, 6, and 7 and parts of Chapters 8, 10, 12, and 13 in a dual-level course entitled *Optimal Design: Theory with Practice*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his or her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. The book can also be used, with some supplementary material, for a second course on engineering optimization or optimum design or structural optimization. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

The book consists of thirteen chapters and two appendices. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn–Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and postoptimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5 through 7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and in-

direct methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems with mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, geometric, and dynamic programming. The theory and applications of calculus of variations, optimal control theory, multiple objective optimization, optimality criteria methods, genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy system optimization are discussed briefly in Chapter 12. The various approximation techniques used to speed up the convergence of practical mechanical and structural optimization problems are outlined in Chapter 13. Appendix A presents the definitions and properties of convex and concave functions. Finally, a brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B.

ACKNOWLEDGMENTS

I wish to thank my wife, Kamala, and daughters, Sridevi and Shobha, for their patience, understanding, encouragement, and support in preparing the manuscript.

S. S. RAO

March 1995

ENGINEERING OPTIMIZATION

**Theory and Practice
Third Edition**

SINGIRESU S. RAO

School of Mechanical Engineering
Purdue University
West Lafayette, Indiana



A Wiley-Interscience Publication

John Wiley & Sons, Inc.

New York • Chichester • Brisbane • Toronto • Singapore

This text is printed on acid-free paper.

Copyright © 1996 by John Wiley & Sons, Inc., Wiley Eastern Limited, Publishers, and New Age International Publishers, Ltd.

All rights reserved. Published simultaneously in Canada.

Reproduction or translation of any part of this work beyond that permitted by Section 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful. Requests for permission or further information should be addressed to the Permissions Department, John Wiley & Sons, Inc.

This publication is designed to provide accurate and authoritative information in regard to the subject matter covered. It is sold with the understanding that the publisher is not engaged in rendering professional services. If legal, accounting, medical, psychological, or any other expert assistance is required, the services of a competent professional person should be sought.

Library of Congress Cataloging in Publication Data:

ISBN 0-471-55034-5

Printed in the United States of America

10 9 8 7 6 5 4

PREFACE

The ever-increasing demand on engineers to lower production costs to withstand competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products both economically and efficiently. Optimization techniques, having reached a degree of maturity over the past several years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems being solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a simple manner. Essential proofs and explanations of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using nonlinear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. This book presents the theory and applications of all optimization techniques in a comprehensive manner. Some of the recently developed methods of optimization, such as genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy optimization, are also discussed in the book.

A large number of solved examples, review questions, problems, figures, and references are included to enhance the presentation of the material. Al-

though emphasis is placed on engineering design problems, examples and problems are taken from several fields of engineering to make the subject appealing to all branches of engineering.

This book can be used either at the junior/senior or first-year-graduate-level optimum design or engineering optimization courses. At Purdue University, I cover Chapters 1, 2, 3, 5, 6, and 7 and parts of Chapters 8, 10, 12, and 13 in a dual-level course entitled *Optimal Design: Theory with Practice*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his or her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. The book can also be used, with some supplementary material, for a second course on engineering optimization or optimum design or structural optimization. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

The book consists of thirteen chapters and two appendices. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn-Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and postoptimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5 through 7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and in-

direct methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems with mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, geometric, and dynamic programming. The theory and applications of calculus of variations, optimal control theory, multiple objective optimization, optimality criteria methods, genetic algorithms, simulated annealing, neural-network-based methods, and fuzzy system optimization are discussed briefly in Chapter 12. The various approximation techniques used to speed up the convergence of practical mechanical and structural optimization problems are outlined in Chapter 13. Appendix A presents the definitions and properties of convex and concave functions. Finally, a brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B.

ACKNOWLEDGMENTS

I wish to thank my wife, Kamala, and daughters, Sridevi and Shobha, for their patience, understanding, encouragement, and support in preparing the manuscript.

S. S. RAO

March 1995

Contents

<i>Preface</i>	vii
<i>Acknowledgments</i>	xi
1. Introduction to Optimization	1
1.1 Introduction	1
1.2 Historical Development	3
1.3 Engineering Applications of Optimization	4
1.4 Statement of an Optimization Problem	5
1.4.1 Design Vector	6
1.4.2 Design Constraints	7
1.4.3 Constraint Surface	8
1.4.4 Objective Function	9
1.4.5 Objective Function Surfaces	10
1.5 Classification of Optimization Problems	15
1.5.1 Classification Based on the Existence of Constraints	15
1.5.2 Classification Based on the Nature of the Design Variables	15
1.5.3 Classification Based on the Physical Structure of the Problem	17
1.5.4 Classification Based on the Nature of the Equations Involved	20

1.5.5	Classification Based on the Permissible Values of the Design Variables	31
1.5.6	Classification Based on the Deterministic Nature of the Variables	32
1.5.7	Classification Based on the Separability of the Functions	34
1.5.8	Classification Based on the Number of Objective Functions	36
1.6	Optimization Techniques	38
1.7	Engineering Optimization Literature	39
	References and Bibliography	40
	Review Questions	44
	Problems	46
2.	Classical Optimization Techniques	65
2.1	Introduction	65
2.2	Single-Variable Optimization	65
2.3	Multivariable Optimization with No Constraints	71
2.3.1	Semidefinite Case	77
2.3.2	Saddle Point	77
2.4	Multivariable Optimization with Equality Constraints	80
2.4.1	Solution by Direct Substitution	80
2.4.2	Solution by the Method of Constrained Variation	82
2.4.3.	Solution by the Method of Lagrange Multipliers	91
2.5	Multivariable Optimization with Inequality Constraints	100
2.5.1	Kuhn-Tucker Conditions	105
2.5.2	Constraint Qualification	105
2.6	Convex Programming Problem	112

References and Bibliography	112
Review Questions	113
Problems	114
3. Linear Programming I: Simplex Method	129
3.1 Introduction	129
3.2 Applications of Linear Programming	130
3.3 Standard Form of a Linear Programming Problem	132
3.4 Geometry of Linear Programming Problems	135
3.5 Definitions and Theorems	139
3.6 Solution of a System of Linear Simultaneous Equations	146
3.7 Pivotal Reduction of a General System of Equations	148
3.8 Motivation of the Simplex Method	152
3.9 Simplex Algorithm	153
3.9.1 Identifying an Optimal Point	154
3.9.2 Improving a Nonoptimal Basic Feasible Solution	154
3.10 Two Phases of the Simplex Method	164
References and Bibliography	172
Review Questions	172
Problems	174
4. Linear Programming II: Additional Topics and Extensions	193
4.1 Introduction	193
4.2 Revised Simplex Method	194
4.3 Duality in Linear Programming	210
4.3.1 Symmetric Primal-Dual Relations	211
4.3.2 General Primal-Dual Relations	211

4.3.3	Primal-Dual Relations When the Primal Is in Standard Form	212
4.3.4	Duality Theorems	214
4.3.5	Dual Simplex Method	214
4.4	Decomposition Principle	219
4.5	Sensitivity or Postoptimality Analysis	228
4.5.1	Changes in the Right-Hand-Side Constants b_i	229
4.5.2	Changes in the Cost Coefficients c_j	235
4.5.3	Addition of New Variables	237
4.5.4	Changes in the Constraint Coefficients a_{ij}	238
4.5.5	Addition of Constraints	241
4.6	Transportation Problem	243
4.7	Karmarkar's Method	246
4.7.1	Statement of the Problem	248
4.7.2	Conversion of an LP Problem into the Required Form	248
4.7.3	Algorithm	251
4.8	Quadratic Programming	254
	References and Bibliography	261
	Review Questions	262
	Problems	263
5.	Nonlinear Programming I: One-Dimensional Minimization Methods	272
5.1	Introduction	272
5.2	Unimodal Function	278
	<i>Elimination Methods</i>	279
5.3	Unrestricted Search	279
5.3.1	Search with Fixed Step Size	279
5.3.2	Search with Accelerated Step Size	280

5.4	Exhaustive Search	281
5.5	Dichotomous Search	283
5.6	Interval Halving Method	286
5.7	Fibonacci Method	289
5.8	Golden Section Method	296
5.9	Comparison of Elimination Methods	298
	<i>Interpolation Methods</i>	299
5.10	Quadratic Interpolation Method	301
5.11	Cubic Interpolation Method	308
5.12	Direct Root Methods	316
	5.12.1 Newton Method	316
	5.12.2 Quasi-Newton Method	319
	5.12.3 Secant Method	321
5.13	Practical Considerations	324
	5.13.1 How to Make the Methods Efficient and More Reliable	324
	5.13.2 Implementation in Multivariable Optimization Problems	325
	5.13.3 Comparison of Methods	325
	References and Bibliography	326
	Review Questions	326
	Problems	327
6.	Nonlinear Programming II: Unconstrained Optimization Techniques	333
6.1	Introduction	333
	6.1.1 Classification of Unconstrained Minimization Methods	336
	6.1.2 General Approach	337
	6.1.3 Rate of Convergence	337
	6.1.4 Scaling of Design Variables	339

<i>Direct Search Methods</i>	343
6.2 Random Search Methods	343
6.2.1 Random Jumping Method	343
6.2.2 Random Walk Method	345
6.2.3 Random Walk Method with Direction Exploitation	347
6.3 Grid Search Method	348
6.4 Univariate Method	350
6.5 Pattern Directions	353
6.6 Hooke and Jeeves' Method	354
6.7 Powell's Method	357
6.7.1 Conjugate Directions	358
6.7.2 Algorithm	362
6.8 Rosenbrock's Method of Rotating Coordinates	368
6.9 Simplex Method	368
6.9.1 Reflection	369
6.9.2 Expansion	372
6.9.3 Contraction	373
<i>Indirect Search (Descent) Methods</i>	376
6.10 Gradient of a Function	376
6.10.1 Evaluation of the Gradient	379
6.10.2 Rate of Change of a Function Along a Direction	380
6.11 Steepest Descent (Cauchy) Method	381
6.12 Conjugate Gradient (Fletcher-Reeves) Method	383
6.12.1 Development of the Fletcher-Reeves Method	384
6.12.2 Fletcher-Reeves Method	386
6.13 Newton's Method	389
6.14 Marquardt Method	392
6.15 Quasi-Newton Methods	394

6.15.1 Rank 1 Updates	395
6.15.2 Rank 2 Updates	397
6.16 Davidon-Fletcher-Powell Method	399
6.17 Broydon-Fletcher-Goldfarb-Shanno Method	405
6.18 Test Functions	408
References and Bibliography	411
Review Questions	413
Problems	415
7. Nonlinear Programming III: Constrained Optimization Techniques	428
7.1 Introduction	428
7.2 Characteristics of a Constrained Problem	428
<i>Direct Methods</i>	432
7.3 Random Search Methods	432
7.4 Complex Method	433
7.5 Sequential Linear Programming	436
7.6 Basic Approach in the Methods of Feasible Directions	443
7.7 Zoutendijk's Method of Feasible Directions	444
7.7.1 Direction-Finding Problem	446
7.7.2 Determination of Step Length	449
7.7.3 Termination Criteria	452
7.8 Rosen's Gradient Projection Method	455
7.8.1 Determination of Step Length	459
7.9 Generalized Reduced Gradient Method	465
7.10 Sequential Quadratic Programming	477
7.10.1 Derivation	477
7.10.2 Solution Procedure	480
<i>Indirect Methods</i>	485
7.11 Transformation Techniques	485

7.12	Basic Approach of the Penalty Function Method	487
7.13	Interior Penalty Function Method	489
7.14	Convex Programming Problem	501
7.15	Exterior Penalty Function Method	502
7.16	Extrapolation Technique in the Interior Penalty Function Method	507
7.16.1	Extrapolation of the Design Vector \mathbf{X}	508
7.16.2	Extrapolation of the Function f	510
7.17	Extended Interior Penalty Function Methods	512
7.17.1	Linear Extended Penalty Function Method	512
7.17.2	Quadratic Extended Penalty Function Method	513
7.18	Penalty Function Method for Problems with Mixed Equality and Inequality Constraints	515
7.18.1	Interior Penalty Function Method	515
7.18.2	Exterior Penalty Function Method	517
7.19	Penalty Function Method for Parametric Constraints	517
7.19.1	Parametric Constraint	517
7.19.2	Handling Parametric Constraints	519
7.20	Augmented Lagrange Multiplier Method	521
7.20.1	Equality-Constrained Problems	521
7.20.2	Inequality-Constrained Problems	523
7.20.3	Mixed Equality-Inequality Constrained Problems	525
7.21	Checking Convergence of Constrained Optimization Problems	527
7.21.1	Perturbing the Design Vector	527
7.21.2	Testing the Kuhn-Tucker Conditions	528

7.22	Test Problems	529
7.22.1	Design of a Three-Bar Truss	530
7.22.2	Design of a Twenty-Five-Bar Space Truss	531
7.22.3	Welded Beam Design	534
7.22.4	Speed Reducer (Gear Train) Design	536
7.22.5	Heat Exchanger Design	537
	References and Bibliography	538
	Review Questions	540
	Problems	543
8.	Geometric Programming	556
8.1	Introduction	556
8.2	Posynomial	556
8.3	Unconstrained Minimization Problem	557
8.4	Solution of an Unconstrained Geometric Programming Problem Using Differential Calculus	558
8.5	Solution of an Unconstrained Geometric Programming Problem Using Arithmetic-Geometric Inequality	566
8.6	Primal-Dual Relationship and Sufficiency Conditions in the Unconstrained Case	567
8.7	Constrained Minimization	575
8.8	Solution of a Constrained Geometric Programming Problem	576
8.9	Primal and Dual Programs in the Case of Less-Than Inequalities	577
8.10	Geometric Programming with Mixed Inequality Constraints	585
8.11	Complementary Geometric Programming	588
8.12	Applications of Geometric Programming	594

References and Bibliography	609
Review Questions	611
Problems	612
9. Dynamic Programming	616
9.1 Introduction	616
9.2 Multistage Decision Processes	617
9.2.1 Definition and Examples	617
9.2.2 Representation of a Multistage Decision Process	618
9.2.3 Conversion of a Nonserial System to a Serial System	620
9.2.4 Types of Multistage Decision Problems	621
9.3 Concept of Suboptimization and the Principle of Optimality	622
9.4 Computational Procedure in Dynamic Programming	626
9.5 Example Illustrating the Calculus Method of Solution	630
9.6 Example Illustrating the Tabular Method of Solution	635
9.7 Conversion of a Final Value Problem into an Initial Value Problem	641
9.8 Linear Programming as a Case of Dynamic Programming	644
9.9 Continuous Dynamic Programming	649
9.10 Additional Applications	653
9.10.1 Design of Continuous Beams	653
9.10.2 Optimal Layout (Geometry) of a Truss	654
9.10.3 Optimal Design of a Gear Train	655
9.10.4 Design of a Minimum-Cost Drainage System	656

References and Bibliography	658
Review Questions	659
Problems	660
10. Integer Programming	667
10.1 Introduction	667
<i>Integer Linear Programming</i>	668
10.2 Graphical Representation	668
10.3 Gomory's Cutting Plane Method	670
10.3.1 Concept of a Cutting Plane	670
10.3.2 Gomory's Method for All-Integer Programming Problems	672
10.3.3 Gomory's Method for Mixed-Integer Programming Problems	679
10.4 Balas' Algorithm for Zero-One Programming Problems	685
<i>Integer Nonlinear Programming</i>	687
10.5 Integer Polynomial Programming	687
10.5.1 Representation of an Integer Variable by an Equivalent System of Binary Variables	688
10.5.2 Conversion of a Zero-One Polynomial Programming Problem into a Zero-One LP Problem	689
10.6 Branch-and-Bound Method	690
10.7 Sequential Linear Discrete Programming	697
10.8 Generalized Penalty Function Method	701
References and Bibliography	707
Review Questions	708
Problems	709

11. Stochastic Programming	715
11.1 Introduction	715
11.2 Basic Concepts of Probability Theory	716
11.2.1 Definition of Probability	716
11.2.2 Random Variables and Probability Density Functions	717
11.2.3 Mean and Standard Deviation	719
11.2.4 Function of a Random Variable	722
11.2.5 Jointly Distributed Random Variables	723
11.2.6 Covariance and Correlation	724
11.2.7 Functions of Several Random Variables	725
11.2.8 Probability Distributions	727
11.2.9 Central Limit Theorem	732
11.3 Stochastic Linear Programming	732
11.4 Stochastic Nonlinear Programming	738
11.4.1 Objective Function	738
11.4.2 Constraints	739
11.5 Stochastic Geometric Programming	746
11.6 Stochastic Dynamic Programming	748
11.6.1 Optimality Criterion	748
11.6.2 Multistage Optimization	749
11.6.3 Stochastic Nature of the Optimum Decisions	753
References and Bibliography	758
Review Questions	759
Problems	761
12. Further Topics in Optimization	768
12.1 Introduction	768

12.2	Separable Programming	769
12.2.1	Transformation of a Nonlinear Function to Separable Form	770
12.2.2	Piecewise Linear Approximation of a Nonlinear Function	772
12.2.3	Formulation of a Separable Nonlinear Programming Problem	774
12.3	Multiobjective Optimization	779
12.3.1	Utility Function Method	780
12.3.2	Inverted Utility Function Method	781
12.3.3	Global Criterion Method	781
12.3.4	Bounded Objective Function Method	781
12.3.5	Lexicographic Method	782
12.3.6	Goal Programming Method	782
12.4	Calculus of Variations	783
12.4.1	Introduction	783
12.4.2	Problem of Calculus of Variations	784
12.4.3	Lagrange Multipliers and Constraints	791
12.4.4	Generalization	795
12.5	Optimal Control Theory	795
12.5.1	Necessary Conditions for Optimal Control	796
12.5.2	Necessary Conditions for a General Problem	799
12.6	Optimality Criteria Methods	800
12.6.1	Optimality Criteria with a Single Displacement Constraint	801
12.6.2	Optimality Criteria with Multiple Displacement Constraints	802
12.6.3	Reciprocal Approximations	803

12.7	Genetic Algorithms	806
12.7.1	Introduction	806
12.7.2	Representation of Design Variables	808
12.7.3	Representation of Objective Function and Constraints	809
12.7.4	Genetic Operators	810
12.7.5	Numerical Results	811
12.8	Simulated Annealing	811
12.9	Neural-Network-Based Optimization	814
12.10	Optimization of Fuzzy Systems	818
12.10.1	Fuzzy Set Theory	818
12.10.2	Optimization of Fuzzy Systems	821
12.10.3	Computational Procedure	823
	References and Bibliography	824
	Review Questions	827
	Problems	829
13.	Practical Aspects of Optimization	836
13.1	Introduction	836
13.2	Reduction of Size of an Optimization Problem	836
13.2.1	Reduced Basis Technique	836
13.2.2	Design Variable Linking Technique	837
13.3	Fast Reanalysis Techniques	839
13.3.1	Incremental Response Approach	839
13.3.2	Basis Vector Approach	845
13.4	Derivatives of Static Displacements and Stresses	847
13.5	Derivatives of Eigenvalues and Eigenvectors	848
13.5.1	Derivatives of λ_j	848
13.5.2	Derivatives of \mathbf{Y}_j	849
13.6	Derivatives of Transient Response	851

13.7	Sensitivity of Optimum Solution to Problem	
	Parameters	854
13.7.1	Sensitivity Equations Using Kuhn-Tucker	
	Conditions	854
13.7.2	Sensitivity Equations Using the Concept of	
	Feasible Direction	857
13.8	Multilevel Optimization	858
13.8.1	Basic Idea	858
13.8.2	Method	859
13.9	Parallel Processing	864
	References and Bibliography	867
	Review Questions	868
	Problems	869
Appendices	876
	Appendix A: Convex and Concave Functions	876
	Appendix B: Some Computational Aspects of Optimization	882
Answers to Selected Problems	888
Index	895